

# Falling over or Sliding down?

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## Problem

- In animations, frictional contact models have been widely used in many areas like rigid body collision and soft body deformation. Given a simple geometric method to construct the difference between friction and non-friction at the in-contact moment.

## Previous work

- Staggered sequence of projections
  - Velocity-level
  - Friction with contact impulses  
[Kaufman, Sueda, James and Pai 2008]
- Convex optimization based algorithm
  - Nonpenetration  
[Drumwright and Shell 2009]
- Interpenetration volume constraints
  - Tangent space basis
  - Volume-based contact constraints  
[Allard et al. 2010]
- Nonsmooth Newton algorithm
  - Simulation thin elastic rods  
[Descoubes, Cadoux, Daviet and Acary 2011]

## Challenges

- The traditional methods put Coulomb friction condition into contact function and use different kinds of iterative LCP solvers to get a global or local result. The complexity of the algorithm becomes the main bottleneck for the frictional contact model.

## Our method

- We use the FEM to calculate the deformation of soft bodies with different materials and give a simple geometry method to represent the friction.

## Compare different methods

- Standard equilibrium equations of dynamic deformation

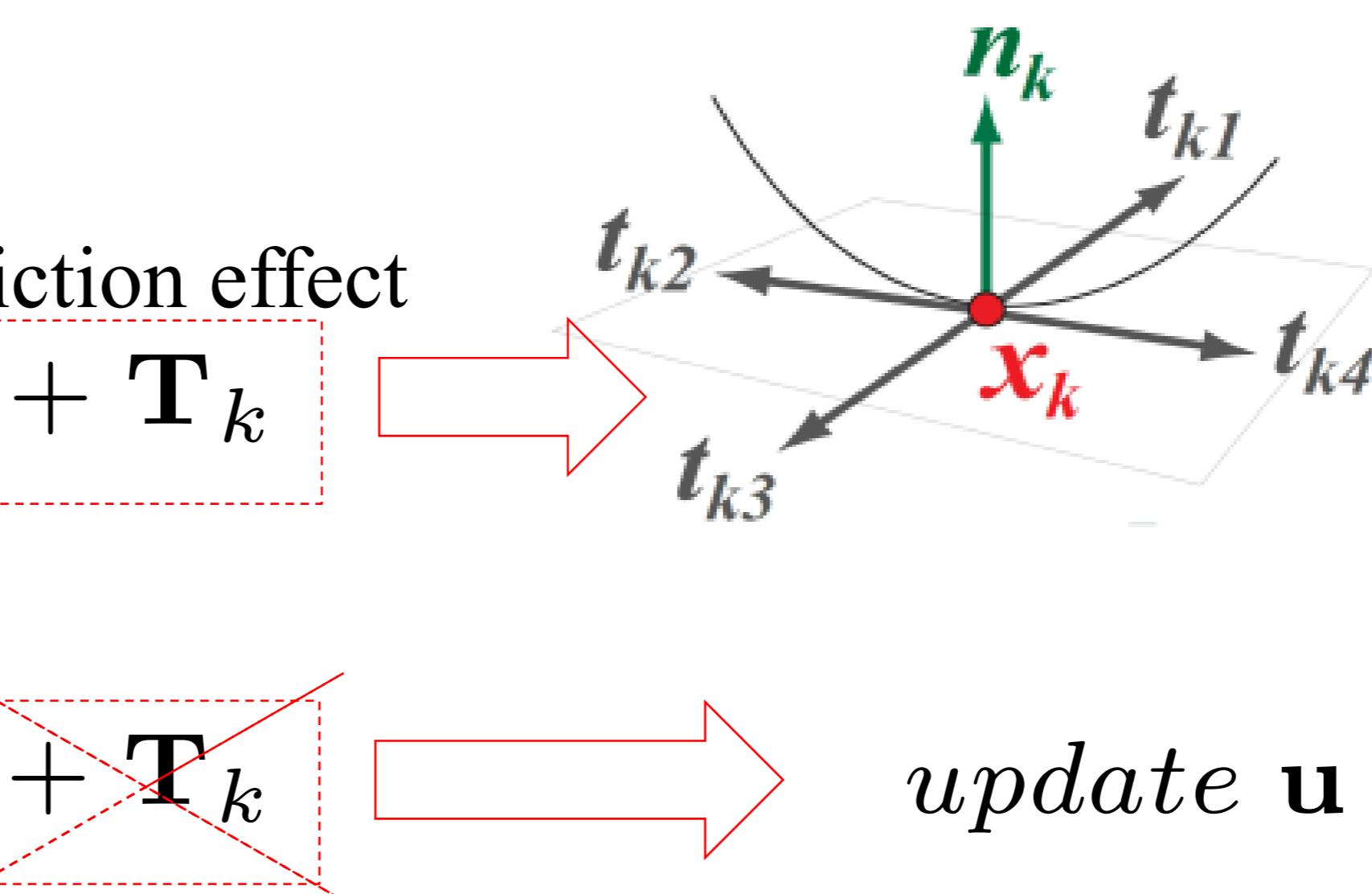
$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{D}\dot{\mathbf{u}} = \mathbf{r} - \mathbf{f}(\mathbf{u})$$

- Traditional methods with the friction effect

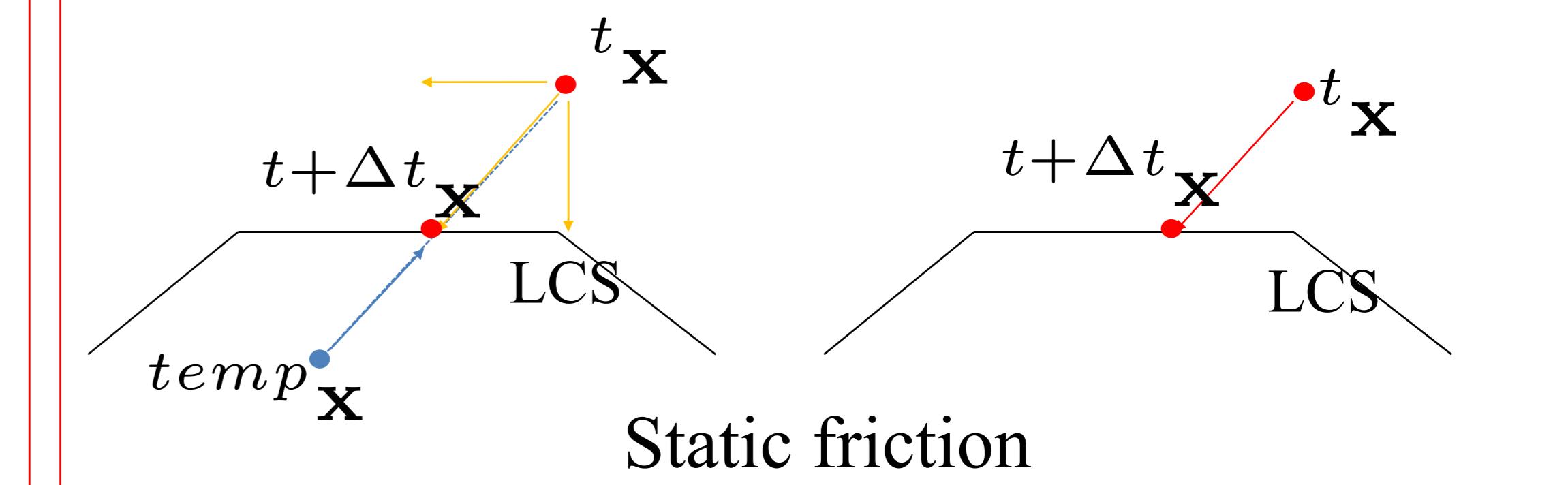
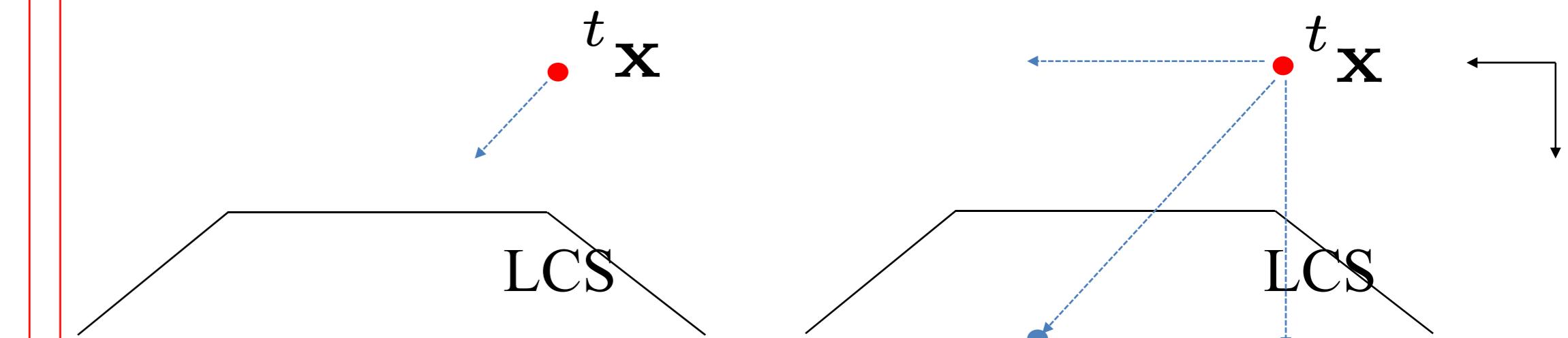
$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{D}\dot{\mathbf{u}} = \mathbf{r} - \mathbf{f}(\mathbf{u}) + \mathbf{T}_k$$

- Our method

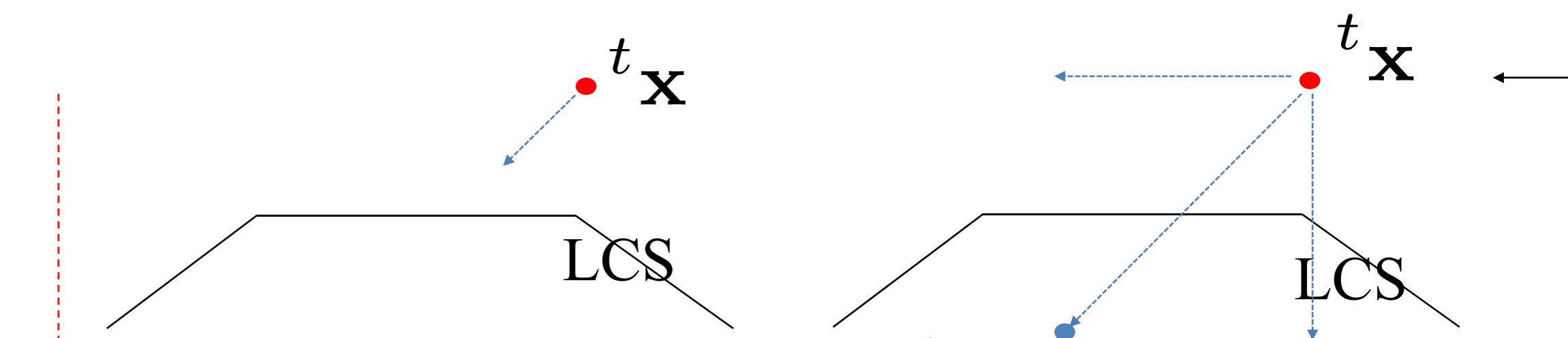
$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{D}\dot{\mathbf{u}} = \mathbf{r} - \mathbf{f}(\mathbf{u}) + \mathbf{T}_k$$



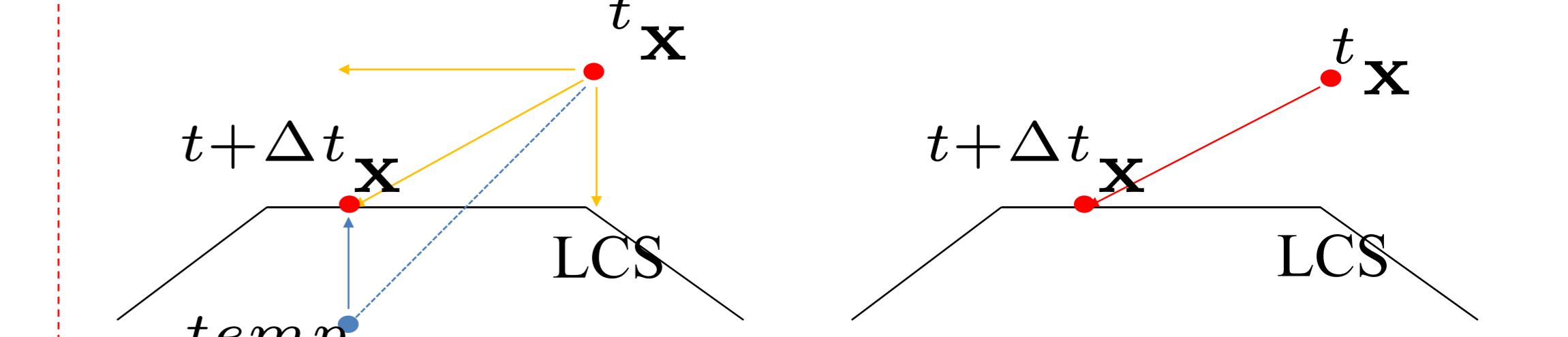
## Process in different frictional modes



Static friction



Non-friction



Sliding-friction

## Update the displacement with friction effect

$${}^{t+\Delta t}\mathbf{u} = {}^t\mathbf{u} + ({}^{t+\Delta t}\mathbf{x} - {}^t\mathbf{x}) = {}^t\mathbf{u} + [u_x, u_y, u_z]^T$$

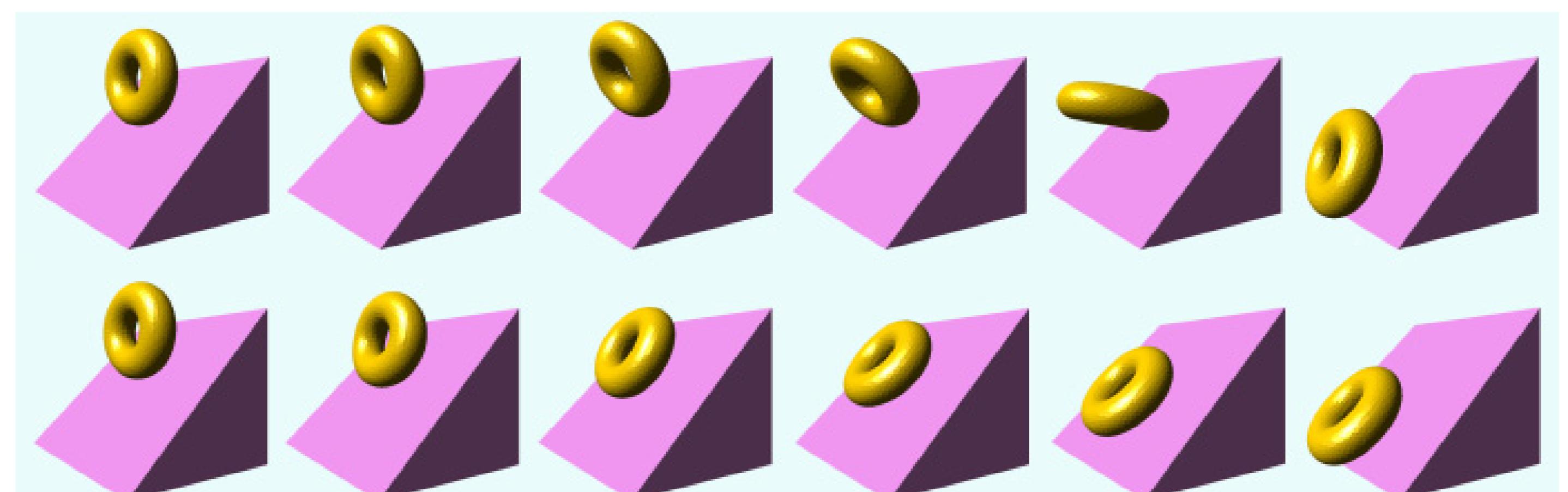
$$\text{We define } {}^{temp}\mathbf{x} - {}^t\mathbf{x} = [\hat{u}_x, \hat{u}_y, \hat{u}_z]^T$$

$u_y$  = distance from  ${}^t\mathbf{x}$  to LCS

$$(u_y/\hat{u}_y)\hat{u}_x = u_{xout} \leq u_x \leq u_{xin} = \hat{u}_x$$

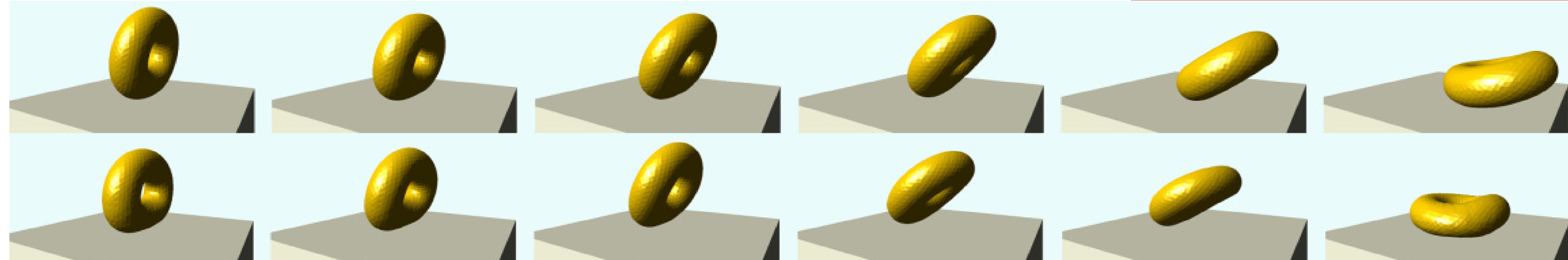
$$(u_y/\hat{u}_y)\hat{u}_z = u_{zout} \leq u_z \leq u_{zin} = \hat{u}_z$$

## Results



Static friction

Sliding friction



Falling over

Sliding down